

Questions “Applied Geometry, Part II”

This list of question is to be changed every once in a while. Please download the most recent list!

I. Questions about “Perspective”

1. What is a “Perspective”?

Perspectives (central views) are the result of a central projection from an eye point to the image plane. This serves to simulate the impression of natural sight.

2. When was perspective drawing invented?

We know (few) examples of perspective drawings from the ancient Greeks and Romans. Systematically, perspective was developed in the Renaissance (Filipo Brunelleschi, Piero della Francesca, Paolo Ucello, Albrecht Drer et al.)

3. What is the perspective image of a straight line? (Proof!) We have to map all points of the straight line. The manifold of the projection rays is a plane. This plane has to be intersected with the image plane. The result is a straight line.

4. What is a vanishing point?

Each point of a straight line has in image point in the image plane. This is also true for the point at infinity. Its image is the vanishing point. Parallels have the point at infinity in common. They thus have the same vanishing point. The principal vanishing point is the image of all normals to the image plane.

5. What is a vanishing line?

Each point of a plane has an image point. In general, all image points fulfil the entire drawing plane. Each plane has a line at infinity where all the points at infinity lie. The image of this line at infinity is called the vanishing line. Parallel planes have the same vanishing lines. The vanishing line of all horizontal planes is the horizon.

6. What is the image of a straight line on our retina?

Our retina is spherically curved. When we intersect the projection plane of the straight line with the sphere, we get (part of a) circle. Our brain has learned to interpret such circular curves as “straight lines”.

7. What is the perspective image of a circle / of a conic section?

In both cases we have to intersect the quadratic projection cone through the center with the image plane, and we thus always obtain a conic circle.

8. What is the silhouette of a sphere in perspective? Discuss the “hyperbolic case”

The projection cone is a cone of revolution. Its intersection with the image plane is again a conic section. A hyperbola only occurs, when the vanishing plane (the plane through the eye point parallel to the image plane) intersects the sphere.

9. Why do we expect the silhouette of a sphere to be a circle?

If possible, we tend to look at the sphere’s center. Then the image plane is perpendicular to the axis of the projection plane, and the silhouette is a circle.

10. What is a primary and what is a secondary perspective?

When we look at an object (or, likewise, take a picture of it), we have the primary situation. When we later on look at the (planar) image, we deal with another (secondary) projection. In order to get a comparable impression of the object, primary and secondary projection have to “fit together”: In the ideal case, the viewing angles are identical.

11. When should we use a wide-angle lens, and when a telescopic lens?

When we need a perspective that is observer from a close distance (large viewing angles in the secondary perspective), the viewing angles in the primary perspective also have to be large (wide angle). accordingly, we use a telescopic lens (small viewing angles) when the image is either depicted small or the viewer has a greater distance from the image.

12. What is the “Gaussian collineation”?

Due to the rules of optics, points in 3-dimensional object space are depicted to virtual points “behind the lens” that also lie in 3-space. Real and virtual objects correspond one to one by means of a collineation (the name “collineation” already indicates that straight lines correspond to straight lines). The two-dimensional image (“photo”) is a cross-section of the 3-dimensional virtual image. Be prepared to make a sketch!!

13. What is the major rule for underwater photography?

“If the picture is not good enough, you were not close enough”. Thus, use wide-angle lenses (or macro lenses plus flash gun). The reason for this is that colors “get lost” very quickly under water (starting with red). Additionally, visibility is rarely more than 15-20 meters.

14. What is a “curved perspective”, what is a “fisheye perspective”?

When we project on curved surfaces (e.g., the spherical retina), we usually have to flatten the image for presentation. Then – in general – straight lines do not appear straight any more. Panoramic views are projections of space onto a (usually vertical) cylinder of revolution. Other curved perspectives are produced by ultra wide-angle lenses called “fisheye lenses”. In such perspectives, straight lines only appear straight if the straight line that is to be depicted intersects the optical axis.

II. Questions about Kinematics (Geometry of Motion)

1. What is a constrained motion? Examples in the plane and in space?

When we move a rigid system in the plane or in space in prescribed manner, we speak of a constrained motion. E.g., if we move two points of a rigid two-dimensional system along straight lines, the system fulfils an elliptic motion (every point of the system will have an ellipse as orbit). The motion of the Earth is also constrained, although there are no guiding lines or similar. The gravity is responsible for that.

2. How is an instantaneous pole?

When a rigid system is moving arbitrarily, its “instantaneous” (momentary) motion is always a rotation. The rotation center is called instantaneous pole. It has momentary velocity zero. All other points have a velocity that is proportional to the distance from the pole. The path normals of all points contain the pole.

3. How is an elliptic motion defined?

A rod of constant length is moved such that the end points move on straight lines. These lines can be but need not be perpendicular. Each point that is rigidly connected with the rod has an elliptic path curve.

4. How else can we induce an elliptic motion?

If a circle rolls inside a circle of double diameter, this also determines an elliptic motion. The circles are called Kardan circles.

5. What is spherical kinematics?

When we have a constrained motion where every path curve lies on a sphere, we speak of spherical kinematics. The most common case is that two rotations about two *intersecting* axes are composed. Examples: “Mixer”, Kardan joints.

6. Why is the Earth’s motion a task of space kinematics?

The motion is quite complicated, since the Earth is moving on an Ellipse with variable speed. Simplified spoken, however, we deal with a rotation about the sun. Additionally, the Earth spins about a *skew* axis (not intersecting!).