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★**The universe of quadrics.**

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Given that quadrics dwell in the memory of most mathematicians just as a distant memory of a marginal topic learned in linear algebra, and given the vast literature on them which is in danger of being irrevocably forgotten, the authors of this marvelous book take upon themselves the task of preserving a good chunk of the knowledge about them for the present and for future generations. Given the enormous wealth of results it contains, it is not possible to give a detailed survey of its contents. It is clearly a labor of love, and if the subject has any chance of gaining readers, then this book is its best chance, given not only the care with which everything is presented, in a self-contained manner, but also the wealth of stunning multi-colored figures of the highest quality.

It starts with quadrics in Euclidean 3-space, presenting the main properties of ellipsoids, hyperboloids, and paraboloids. One learns already in this first chapter charming properties, such as "the sides of a non-planar quadrilateral are located on a one-sheeted hyperboloid of revolution if, and only if, an alternating sum of its side lengths vanishes"; "any two skew lines s_1 and s_2 are respective axes of two one-sheeted hyperboloids of revolution which contact each other along a common generator c"; or "given a skew quadrilateral in $\mathbb{E}^3$, there exists a unique hyperbolic paraboloid passing through the four sides."

The next two chapters focus on the linear algebraic approach to quadrics, projective and affine quadrics, in which 3- and n-dimensional projective space, together with correlations and polarities, are introduced, including a section devoted to projective models of non-Euclidean geometries.

Next come: a chapter on pencils of quadrics, one on cubic and quartic space curves as intersections of quadrics, where we also find Viviani's curve (or Viviani's window), one on confocal quadrics (including Ivory's theorem and the string construction of quadrics), one on differential geometric aspects of quadrics, one on line geometry (which is another forgotten continent) and its connection to quadrics, kinematics, special properties, and generalizations of quadrics. *Victor V. Pambuccian*